

LAUNCHING OF THE HE<sub>11</sub> SURFACE WAVE MODE BY AN  
ELECTRIC DIPOLE IMBEDDED IN A DIELECTRIC ROD

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Recently, there is much renewed interest in dielectric surface waveguides in view of their potential application in communications at the millimetric and optical frequencies. Many of the theoretical and practical problems involved were discussed in an informative survey by Kao<sup>1</sup>. Among them, the excitation of surface waves on these waveguiding structures is, evidently, of much importance. One of the simplest structures, which is, nevertheless, of great practical utility, is the circular dielectric rod. The excitation of circularly symmetric surface waves on a dielectric rod by an elementary source, for example, a magnetic current ring, has been investigated previously<sup>2,3</sup>. However, a similar treatment for the HE<sub>11</sub> dipole mode is not available. The excitation of the HE<sub>11</sub> mode is of great practical importance, since it is the dominant mode and the easiest one to excite in a pure form if a single-mode operation is desired. Consequently, the HE<sub>11</sub> mode has been widely used<sup>1,4</sup>. Snyder<sup>5</sup> used an asymptotic approach in dealing with the excitation of modes on a semi-infinite dielectric rod, since the exact Green's functions for the fields were not known except for the circularly symmetric modes (TM<sub>0m</sub> or TE<sub>0m</sub>). It is the purpose of the present paper to present a theoretical study of the problem of exciting the HE<sub>11</sub> mode. Some preliminary work in this direction was reported in a recent communication by Yip<sup>6</sup>. By applying a technique, which involves expressing the source and fields in a Fourier integral in the  $z$  direction and a Fourier series in the  $\theta$  direction in a cylindrical co-ordinate system ( $\rho, \theta, z$ ) such as has been extensively used by Wait<sup>7</sup>, exact forms for the field solutions (i.e. the Green's functions for HE<sub>11</sub> mode) can be obtained. The solutions are then used to assess the launching efficiency of the HE<sub>11</sub> surface-wave mode. The same technique can also be used to study other cylindrically stratified dielectric structures.

The dielectric rod, characterized by a permittivity of  $\epsilon = \epsilon_0 \epsilon_r$  and a permeability of  $\mu_0$ , is assumed to be lossless and infinitely long with its axis coinciding with the  $z$  axis of a cylindrical co-ordinate system. It has a radius  $a$ , and is surrounded by free space ( $\mu_0, \epsilon_0$ ). A point electric dipole is placed at the origin, but oriented perpendicular to the  $z$  axis. Only the time-harmonic case with time dependence  $\exp(-i\omega t)$  is treated. The current in the dipole can be expressed as follows:

$$\underline{J}_1(\underline{r}, t) = \hat{x} J_1(\underline{r}, t) = \hat{x} J_e \frac{\delta(\rho)}{2\pi\rho} \delta(z) e^{-i\omega t} \quad (1)$$

where  $\hat{x}$  is the unit vector along the  $x$  axis and given by  $\hat{x} = \hat{\rho} \cos \theta - \hat{\theta} \sin \theta$ ,  $\hat{\rho}$  and  $\hat{\theta}$  being the unit vectors in the radial and azimuthal directions respectively.

By the Fourier transform technique, the fields and current inside and outside the rod can be expressed in the following form:

$$f(\rho, \theta, z) = \frac{k_o}{2\pi} \int_{-\infty}^{\infty} F(\rho, \theta, \gamma) e^{ik_o \gamma z} d\gamma \quad (2)$$

$\gamma$  being the normalized axial propagation coefficient. The transformed fields,  $F(\rho, \theta, \gamma)$  are further expressed in a Fourier series.

$$F(\rho, \theta, \gamma) = \sum_{m=-\infty}^{\infty} F_m(\rho, \gamma) e^{im\theta} \quad (3)$$

With the help of Maxwell's equations, solutions are first sought for  $H_z(\rho, \theta, z)$  and  $E_z(\rho, \theta, z)$ , and all the other field components can be expressed in terms of  $H_z$  and  $E_z$ . Examination of the source terms in (1) immediately reveals that only the dipolar terms yielding  $m = 1$  and  $-1$  will be excited. Explicit expressions for the transformed fields can be obtained through the imposition of the boundary conditions which require the continuity of the tangential electric and magnetic fields at the dielectric-air interface.

The actual fields  $H_z(\rho, \theta, z)$  and  $E_z(\rho, \theta, z)$  can be found by Fourier transformation as indicated in (2). The resulting integral expressions for the fields can be evaluated by means of a contour integration. The residues at the real poles of the integrand give rise to surface waves, guided along the dielectric rod. The real poles of the integrand are given by the real roots  $\gamma_s$  of the following dispersion equation:

$$\gamma^2 \left( \frac{1}{v^2} - \frac{1}{u^2} \right)^2 = \left[ \frac{J_1'(u)}{u J_1(u)} - \frac{H_1^{(1)'}(v)}{v H_1^{(1)}(v)} \right] \left[ \frac{\epsilon_r J_1'(u)}{u J_1(u)} - \frac{H_1^{(1)'}(v)}{v H_1^{(1)}(v)} \right] \quad (4)$$

which is the well-known dispersion equation for the  $HE_{11}$  mode on a dielectric rod<sup>8</sup>. In (4),  $u = k_o a \eta$ ,  $v = k_o a \eta_o$ ,  $\eta = \sqrt{\epsilon_r} - \gamma^2$  and  $\eta_o = \sqrt{1 - \gamma^2}$ . The dispersion curves are as shown in Figure 1 for different values of  $\epsilon_r$ . The total surface-wave power  $P_s$  excited by the dipole can be evaluated by integrating the real part of the time-averaged axial Poynting vector across an infinite plane normal to the  $z$  axis. Numerical results for  $P_s$  are presented in Figure 2 for  $\lambda_o = 8$  mm. These results indicate that, for a given dielectric material, there is an optimum rod radius for which  $P_s$  is a maximum.

In addition to the surface waves, the dipole also radiates space waves, which arise from the saddle-point contribution of the integrand. The total radiated power  $P_r$  can be evaluated by integrating the  $\rho$ -component of the Poynting vector across the surface of an infinitely long cylinder with its axis lying on the  $z$  axis and its radius greater than  $a$ . For  $|\gamma| > 1$ , the fields are evanescent. Through the application of Parseval's theorem,  $P_r$  can be obtained by integrating over all effective values of  $\gamma$  i.e.  $|\gamma| < 1$  as follows:

$$P_r = \frac{J_e^2}{2} \int_0^{2\pi} d\theta \left[ 2 \int_0^1 P_1(\gamma, \theta) d\gamma \right] = \frac{J_e^2}{2} R_r \quad (5)$$

where  $R_r$  is defined as the radiation resistance. To assess the amount of power which appears in the space waves,  $P_r$  was computed as a function of  $a$  and  $\epsilon_r$  for  $\lambda_o = 8$  mm.  $P_r$  is found to exhibit an oscillatory behaviour. This phenomenon can be ascribed to the formation of standing waves inside the dielectric rod.

The launching efficiency  $\alpha$  of the surface waves has also been computed. It is found that, with an appropriate selection of  $\epsilon_r$  and  $a$ , surface waves can be excited with efficiencies higher than 80%. In general, for a fixed  $\epsilon_r$  the highest efficiency is achieved at a relatively small radius. This is actually quite a desirable feature both with regard to a single mode operation and the material cost. Thus, for example, at  $\epsilon_r = 2.56$ , the highest efficiency is obtained for  $a = 2$  mm. This yields  $2a/\lambda_o = 0.5$ , which is well below the cutoff value of  $2a/\lambda_o = 0.613$  for the next higher-order axially symmetric modes  $TE_{01}$  and  $TM_{01}$ .<sup>8</sup>

The radiation power patterns of the dipole imbedded in the dielectric rod will also be presented, and the effect of the dielectric rod on the radiation characteristics of the dipole discussed.

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#### Acknowledgement

The authors would like to thank Professor G.W. Farnell for the encouragement of this research. The work reported here is supported by the National Research Council of Canada under grant A-4411.

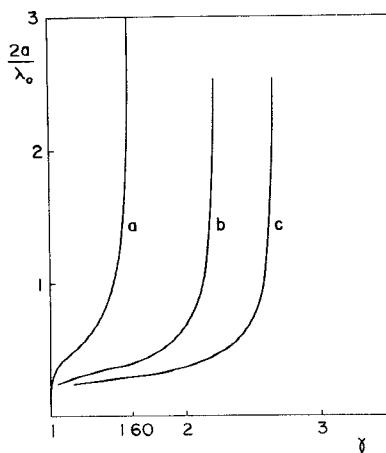


FIG. 1

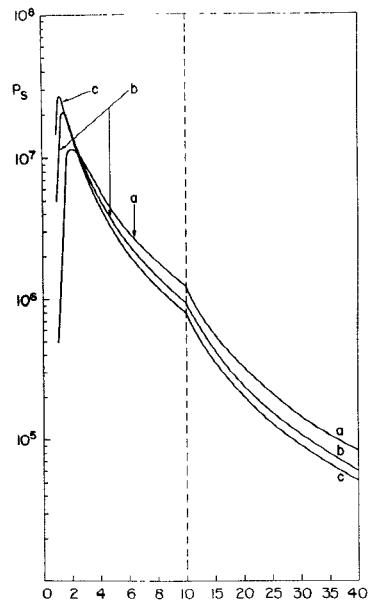


FIG. 2

Figure 1. Dispersion curves for the  $HE_{11}$  mode on a dielectric rod.

(a)  $\epsilon_r = 2.56$ , (b)  $\epsilon_r = 5.0$ ,  
 (c)  $\epsilon_r = 7.12$ .

Figure 2. Surface-wave power for the  $HE_{11}$  mode at  $\lambda_0 = 8$  mm.  $P_s$  as a function of  $a$ . (a)  $\epsilon_r = 2.56$ , (b)  $\epsilon_r = 5.0$ ,  
 (c)  $\epsilon_r = 7.12$ .

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